

## ANNEX B

UNIVERSITY OF MILAN

Public selection for recruiting No.4836 research associate(s) under art.24, paragraph 3.a, of Law No.240/2010 for competition sector: 01/A2 - Geometry and Algebra, (scientific-disciplinary sector: MAT/03-Geometry) at the Department of Mathematics, (announcement published in Official Gazette No.75 of 21 September 2021) - Competition code 4836

## [Luming ZHAO] CURRICULUM VITAE

(N.B. CV MUST BE OF UP TO 30 PAGES AND INCLUDE THE DETAILS CANDIDATES CONSIDER USEFUL FOR THE ASSESSMENT.

ALL THE TITLES INSERTED BELOW ARE JUST EXAMPLES THAT CAN BE REPLACED, CHANGED OR COMPLETED)

PERSONAL DATA (DO NOT INCLUDE YOUR PERSONAL ADDRESS AND LANDLINE OR MOBILE PHONE NUMBER)

SURNAME	ZHAO
NAME	LUMING
DATE OF BIRTH	[ 11, 08, 1992 ]

## QUALIFICATIONS

### DEGREE

(Specify full degree name, University, date, etc.)

- Ph.D. degree on mathematics, Sep.2021  
University of Bordeaux, Bordeaux, France
- master degree on pure mathematics, Jul.2017  
Algant Program, Erasmus Mundus:  
University of Bordeaux, Bordeaux, France and University of Padua, Padua, Italy,
- bachelor degree on pure mathematics, Jul.2015  
Shandong University, Jinan, China

DOCTORAL DEGREE OR EQUIVALENT QUALIFICATION EARNED IN ITALY OR ABROAD / MEDICAL SPECIALISATION DIPLOMA OR EQUIVALENT QUALIFICATION, FOR THE RELEVANT SECTORS, EARNED IN ITALY OR ABROAD

(Specify qualification full name, institution, date, etc.)

Ph.D. degree on pure mathematics- University of Bordeaux, Bordeaux, France, Sep.2017-Sep.2021

**RESEARCH CONTRACTS, RESEARCH FELLOWSHIP CONTRACTS, POSTDOCTORAL SCHOLARSHIPS OR SIMILAR CONTRACTS**

*(Specify, for each contract, university/institution, starting and termination date, etc.)*

ATER contract, University of Bordeaux, Sep.2020-Mar.2021

ATER contract, University of Bordeaux, Sep.2021-Sep.2022

**TEACHING ACTIVITIES AT ITALIAN OR FOREIGN UNIVERSITIES**

*(Specify academic year, university, degree course, number of hours etc.)*

- Algebraic structure 2, 3rd year course in French, exercise session, University of Bordeaux, 2021
- General Mathematics, 1st year course in English, lecture and exercise session, University of Bordeaux, 2021
- Mathematical Tools, 1st year course in French, exercise session, University of Bordeaux, 2021
- Basic Mathematics, 1st year course in English, exercise session, University of Bordeaux, 2020
- Coloring Mathematics, 1st year course in French, exercise session, University of Bordeaux, 2020

**ATTESTED TRAINING OR RESEARCH ACTIVITIES AT QUALIFIED ITALIAN OR FOREIGN INSTITUTIONS**

*(Specify academic year, institution, course, period, etc.)*

Automorphic Forms and Representation Theory Seminar, online talk, Purdue University, September 2021.

La journée de l'EDMI, poster presentation, University of Bordeaux, April 2019

**ATTESTED ACTIVITY IN THE CLINICAL FIELD**

*(Specify date, duration, role, institution where the aid activity was carried out, etc.)*

**IMPLEMENTATION OF PROJECTS**

*(Specify date, project name, etc.)*

**ORGANISATION, SUPERVISION AND COORDINATION OF NATIONAL AND INTERNATIONAL RESEARCH GROUPS, OR PARTICIPATION IN THEM**

*(For each entry, specify year, role, research group, etc.)*

**HOLDING PATENTS**

*(For each patent, specify authors' names, title, classification, patent number, etc.)*

**SPEAKING AT NATIONAL AND INTERNATIONAL CONFERENCES AND CONVENTIONS**

*(Specify conference/convention title, date, etc.)*

**NATIONAL AND INTERNATIONAL AWARDS AND ACCOLADES FOR RESEARCH ACTIVITY**

*(Specify award, date, issuing organisation, etc.)*

**HOLDING A EUROPEAN SPECIALISATION DIPLOMA RECOGNISED BY INTERNATIONAL BOARDS**

*(For those competition sectors for which it is requested)*

*(Specify diploma, date, etc.)*

**QUALIFICATIONS UNDER ART.24, PARAGRAPH 3.a AND 3.b, OF LAW No.240/2010 OF 30 DECEMBER 2010**

*(Specify whether it is a type A or type B contract, University, contract effective date and end date, etc.)*

**SCIENTIFIC PRODUCTION**

**SCIENTIFIC PUBLICATIONS**

*(For each publication, specify the following: authors' names, full title, publisher, date and place of publication, ISBN/ISSN/DOI or equivalent code)*

Luming Zhao. Galois cohomology of p-adic fields and  $(\phi, \tau)$ -modules. General Mathematics [math.GM].  
Université de Bordeaux, 2021. English. (NNT : 2021BORD0217). (tel-03374406)

Date

17 Oct 2021

Place

Talence, France

Luning MAO

# CURRICULUM VITAE

## Personal Information

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- Address: Apartment 129, Building B, 9 Rue Porte-Bonheur, Talence, France
- Birth Date: 1992/08/11
- Family situation: married

## Experience:

- ATER, University of Bordeaux, Bordeaux, France, Sep.2021-present
- ATER, University of Bordeaux, Bordeaux, France, Sep.2020-Mar.2021

## Education Background:

- Ph.D. Mathematics
  - University of Bordeaux, Bordeaux, France, Sep.2017-Sep.2021  
Ph.D. Thesis, "Galois cohomology for  $p$ -adic fields and  $(\varphi, \tau)$ -modules"  
Advisor: Olivier Brinon, University of Bordeaux
- M.Sc. Mathematics, Algant Program, Erasmus Mundus
  - **Master 2** University of Bordeaux, Bordeaux, France, Sep.2016-Jul.2017  
mention Très Bien  
Master Thesis, "Comparison theorems for  $p$ -divisible groups"  
Advisor: Olivier Brinon, University of Bordeaux
  - **Master 1** University of Padua, Padua, Italy, Sep.2015-Jul.2016
- B.Sc. Mathematics,
  - Shandong University, Jinan, China, Sep.2011-Jul.2015
  - AMSS, Chinese Academy of science, Beijing, China, Sep.2014-Jun.2015  
Bachelor Thesis, "Classification of complex algebraic surfaces"  
Advisor: Yichao Tian, AMSS, CAS

## Research Interests

On Arithmetic Geometry, especially on  $p$ -adic Hodge theory.

## Invited Talks

- Automorphic Forms and Representation Theory Seminar, Purdue University, Sep.2021

## **Financial supports, Scholarships**

- Erasmus Mundus scholarship of Alcant Master Program, Padova, Italy and Bordeaux, France.
- Admitted to Hua Loo-Keng Talent Program in math, Shandong University and AMSS, CAS, China.
- Outstanding Student Scholarship, Shandong University, Jinan, China.

## **Teaching**

- Fall 2021: Algebraic structure 2, exercise sessions, 3rd year undergraduate course for math department, in French, University of Bordeaux.
- Fall 2021: General Mathematics, lectures and exercise sessions, 1st year undergraduate course for science department in English, University of Bordeaux.
- Fall 2021: Mathematical Tools, exercise sessions, 1st year undergraduate course for science department in French, University of Bordeaux.
- Fall 2020: Basic Mathematics, exercise sessions, 1st year undergraduate course for science department in English, University of Bordeaux.
- Fall 2021: Coloring Mathematics, exercise sessions, 1st year undergraduate course for science department in French, University of Bordeaux.

## **Languages**

- Chinese (native)
- English (fluent)
- French (sufficient for research and teaching)

# Research statement

Luming ZHAO

October 2021

## 1 Summary of the thesis

Let  $p$  be a prime integer and  $K$  a complete discrete valuation field of characteristic 0 with residue field  $k$  perfect of characteristic  $p$ . Fix  $\overline{K}$  an algebraic closure of  $K$  and denote  $\mathcal{G}_K = \mathrm{Gal}(\overline{K}/K)$ . We are interested in the category  $\mathbf{Rep}_{\mathbf{Q}_p}(\mathcal{G}_K)$  of  $p$ -adic representations of  $\mathcal{G}_K$ . Fix  $\varepsilon = (\zeta_{p^n})_{n \in \mathbf{N}}$  a compatible system of primitive  $p^n$ -th roots of unity, and  $K_\infty = K_\zeta := \bigcup_{n=0}^{\infty} K(\zeta_{p^n})$  be the cyclotomic extension of  $K$ : it is Galois with group  $\Gamma = \mathrm{Gal}(K_\zeta/K)$ , which identifies with an open subgroup of  $\mathbf{Z}_p^\times$  through the cyclotomic character.

To study it, Fontaine defined  $(\varphi, \Gamma)$ -modules over some ring of formal series  $\mathcal{O}_{\mathcal{E}}$ , endowed with commuting actions of  $\Gamma$  and a Frobenius operator  $\varphi$  (that lifts the Frobenius modulo  $p$ ). These are  $\mathcal{O}_{\mathcal{E}}$ -modules  $D$  of finite type that are endowed with a semilinear and commuting actions of  $\Gamma$  and of a Frobenius operator  $\varphi$ . He proved that  $\mathbf{Rep}_{\mathbf{Q}_p}(\mathcal{G}_K)$  is categorically equivalent to the category of étale  $(\varphi, \Gamma)$ -modules (étale means the linearization map of Frobenius  $1 \otimes \varphi : \varphi^* D \rightarrow D$  is an isomorphism). The categorical equivalence implies that we should be able to recover invariants of a  $p$ -adic representation  $V$  in term of the associated  $(\varphi, \Gamma)$ -module.

For Galois cohomology, this was done by Herr: he constructed a complex with three terms, using the  $(\varphi, \Gamma)$ -modules associated to  $V$ , the so-called Herr complex and whose homology computes  $H^i(\mathcal{G}_K, V)$  for all  $i \in \mathbf{N}$ .

There are other natural choices for  $K_\infty$ , for example the extension generated by torsion points of a Lubin-Tate group over a sub-field of  $K$  or the Galois closure of a Kummer extension. Besides these, we are especially interested in the case where  $K_\infty/K$  is a Kummer extension. More precisely, fix a uniformiser  $\pi$  of  $K$  and a compatible system  $\tilde{\pi} = (\pi_n)_{n \in \mathbf{N}}$  of  $p^n$ -th roots of  $\pi$  (*i.e.* such that

$\pi_0 = \pi$  and  $\pi_{n+1}^p = \pi_n$  for all  $n \in \mathbf{N}$ ): the associated Kummer extension  $K_\infty = K_\pi := \bigcup_{n=0}^\infty K(\pi_n)$  is a so-called Breuil-Kisin extension. Indeed, since the work of Breuil (*cf* [3]) and Kisin (*cf* [5]), it has become clear that the Breuil-Kisin extensions are the right extension to study many questions (applications to  $p$ -divisible groups, "integral"  $p$ -adic Hodge theory, the study of the deformations of  $p$ -adic representations, the study of semi-stable representations and crystalline representations, *etc*).

As it can be expected, there are analogues to Herr complex for different choices of  $K_\infty$ : the Lubin-Tate extension case was treated by Aribam and Kwatra (*cf* [1]) and also by Kupferer and Venjakob ([6]), the Kummer-Galois case is treated by Tavares-Ribeiro (*cf* [7]). Basically, I treated the Breuil-Kisin extension case, using Caruso's  $(\varphi, \tau)$ -module theory: this is an analogue of  $(\varphi, \Gamma)$ -module theory using  $K_\infty = K_\pi$ . The main issue in this setting is the fact that  $K_\pi/K$  is not Galois, so that there is no  $\Gamma$  in this context. The idea is, for any  $T \in \mathbf{Rep}_{\mathbf{Z}_p}(\mathcal{G}_K)$ , to consider not only the associated  $\varphi$ -module  $\mathcal{D}(T)$  over  $\mathcal{O}_\mathcal{E}$ , but also the action of a topological generator  $\tau$  of  $\mathrm{Gal}(L/K_\zeta)$  on  $\mathcal{D}(T)_\tau := \mathcal{O}_{\mathcal{E}_\tau} \otimes_{\mathcal{O}_\mathcal{E}} \mathcal{D}(T)$  (where  $\mathcal{E}_\tau$  is a suitable extension of the fraction field  $\mathcal{E}$  of  $\mathcal{O}_\mathcal{E}$  and  $\mathcal{O}_{\mathcal{E}_\tau}$  its ring of integers). Explicitly we have:

**Theorem 1.1.** (*Caruso, [4, §1.3]*) *The functor*

$$\begin{aligned} \mathbf{Rep}_{\mathbf{Z}_p}(\mathcal{G}_K) &\rightarrow \mathbf{Mod}_{\mathcal{O}_\mathcal{E}, \mathcal{O}_{\mathcal{E}_\tau}}(\varphi, \tau) \\ T &\mapsto \mathcal{D}(T) = (\widehat{\mathcal{O}_{\mathcal{E}^{\mathrm{ur}}}} \otimes_{\mathbf{Z}_p} T)^{\mathcal{G}_{K_\pi}} \end{aligned}$$

*with a natural  $\tau$ -semilinear action  $\tau_D$  over  $\mathcal{D}(T)_\tau := \mathcal{O}_{\mathcal{E}_\tau} \otimes_{\mathcal{O}_\mathcal{E}} \mathcal{D}(T)$ , is an equivalence between the category of integral  $p$ -adic representations and that of  $(\varphi, \tau)$ -modules over  $\mathcal{O}_\mathcal{E}$ .*

Given any integral  $p$ -adic representation  $T$ , I then defined a complex  $\mathcal{C}_{\varphi, \tau}(T)$ , using its  $(\varphi, \tau)$ -modules  $(\mathcal{D}(T), \mathcal{D}(T)_\tau) \in \mathbf{Mod}_{\mathcal{O}_\mathcal{E}, \mathcal{O}_{\mathcal{E}_\tau}}(\varphi, \tau)$  associated.

**Definition 1.2.** ([10, Definition 1.1.12]) *Let  $T \in \mathbf{Rep}_{\mathbf{Z}_p}(\mathcal{G}_K)$ , we define a complex  $\mathcal{C}_{\varphi, \tau}(T)$  as follows:*

$$0 \longrightarrow \mathcal{D}(T) \longrightarrow \mathcal{D}(T) \oplus \mathcal{D}(T)_{\tau, 0} \longrightarrow \mathcal{D}(T)_{\tau, 0} \longrightarrow 0$$

$$x \longmapsto ((\varphi - 1)(x), (\tau_D - 1)(x))$$

$$(y, z) \longmapsto (\tau_D - 1)(y) - (\varphi - 1)(z)$$



where the first term is of degree  $-1$  and  $\mathcal{D}(T)_{\tau,0}$  is a subgroup of  $\mathcal{D}(T)_\tau$  defined in terms of the action of  $\Gamma$  and  $\tau$ .

I proved that the homology of the complex computes the Galois cohomology.

**Theorem 1.3.** ([10, Theorem 1.1.13]) *For any  $T \in \mathbf{Rep}_{\mathbf{Z}_p}(\mathcal{G}_K)$  and natural integer  $i$ , there is a canonical and functorial isomorphism*

$$H^i(\mathcal{C}_{\varphi,\tau}(T)) \simeq H^i(\mathcal{G}_K, T).$$

Similarly, if  $V \in \mathbf{Rep}_{\mathbf{Q}_p}(\mathcal{G}_K)$ , we have a canonical and functorial isomorphism  $H^i(\mathcal{C}_{\varphi,\tau}(V)) \simeq H^i(\mathcal{G}_K, V)$  for all  $i$ .

I constructed a morphism between the four terms complex  $\mathcal{C}_{\varphi,\tau,\gamma}$  constructed by Tavares Ribeiro and our complex  $\mathcal{C}_{\varphi,\tau}$ , and proved that it is a quasi-isomorphism when the residue field is finite. This shows that our complex is a refinement of the former one in this case.

I also constructed a  $\psi$ -operator (a left inverse to the Frobenius  $\varphi$ ) and use it to construct other complexes that compute Galois cohomology.

More precisely, the coefficients of  $(\varphi, \tau)$ -modules described above have perfect residue fields, which makes it impossible to define a  $\psi$ -operator on them: it is necessary to work with partially unperfected coefficients. For this reason I constructed firstly a complex  $\mathcal{C}_{\varphi,\tau}^u$  with partially unperfected coefficients and proved it computes Galois cohomology as well. Then I constructed the complex  $\mathcal{C}_{\psi,\tau}^u$  with  $\psi$ -operator and prove that it is quasi-isomorphic to  $\mathcal{C}_{\varphi,\tau}^u$ , hence it computes Galois cohomology.

Based on recent work of Poyeton and Gao-Poyeton, I built overconvergent avatars of the previous complexes, using either the Frobenius operators  $\varphi$  or the  $\psi$ -operator. In particular I constructed complexes  $\mathcal{C}_{\varphi,\tau}^{\dagger,u}$  with overconvergent and partially unperfected coefficients, and similarly the complex  $\mathcal{C}_{\psi,\tau}^{\dagger,u}$  with  $\psi$ -operator, and proved the two complexes are quasi-isomorphic. By refining the overconvergence result of  $(\varphi, \tau)$ -modules to the case of integral representations, I proved the above complexes  $\mathcal{C}_{\varphi,\tau}^{\dagger,u}$  and  $\mathcal{C}_{\psi,\tau}^{\dagger,u}$  compute the cohomology in degrees 0 and 1.

I also constructed a three-terms complex  $\mathcal{C}_{\varphi,N_\nabla}$  from the  $(\varphi, N_\nabla)$ -module (where  $N_\nabla$  is a normalization of  $\log \tau$ ) over the Robba ring, which is associated to the corresponding  $(\varphi, \tau)$ -modules of

$V \in \mathbf{Rep}_{\mathbf{Q}_p}(\mathcal{G}_K)$  over the Robba ring. As  $N_{\nabla}$  is an infinitesimal version of  $\tau$ , the complex cannot compute Galois cohomology of the Galois representation  $V$ , but it almost can: I proved that its homology group  $H^i$  is isomorphic to  $\varinjlim_n H^i(\mathcal{G}_{K_n}, V)$  for  $i \in \{0, 1\}$ , where  $K_n = K(\pi_n)$ .

By relating the  $(\varphi, \tau)$ -module associated to the Tate module of a  $p$ -divisible group over  $\mathcal{O}_K$  with its corresponding Breuil-Kisin module, our complex  $\mathcal{C}_{\varphi, \tau}(\mathfrak{M})$  can compute the Galois cohomology of the dual of the Tate module of a  $p$ -divisible group from its Breuil-Kisin module.

## 2 Research plans

### 2.1 Left questions in the thesis

Among those different versions of Herr complexes I constructed in the thesis, some I only have partial results that I expect to improve.

#### (1) $H^2$ of the complex $\mathcal{C}_{\varphi, \tau}^{u, \dagger}$ and $\mathcal{C}_{\psi, \tau}^{u, \dagger}$

The homology of the complexes with overconvergent and unperfected coefficients  $\mathcal{C}_{\varphi, \tau}^{u, \dagger}$  and  $\mathcal{C}_{\psi, \tau}^{u, \dagger}$  provide Galois cohomology in degrees 0 and 1: I expect they also do in degree 2. I plan to address this, at least in the case where the residue field is finite.

#### (2) $H^2$ of the complex $\mathcal{C}_{\varphi, N_{\nabla}}$

To study it (in the case where the residue field is finite), I constructed a pairing inspired by Tate duality, and plan to use Tate duality to show that it gives the expected  $H^2$ , but I have only proved the non-degeneracy on one side so far. The non-degeneracy on the other side would imply that the complex over Robba rings  $\mathcal{C}_{\varphi, N_{\nabla}}(V)$  calculates  $\varinjlim_n H^i(\mathcal{G}_{K_n}, V)$  also for  $i = 2$ .

#### (3) The stationary of $\varinjlim_n H^i(\mathcal{G}_{K_n}, V)$

We have given an example where  $\varinjlim_n H^i(\mathcal{G}_{K_n}, V)$  is not stationary. It is interesting to study more examples and give a criterion for the limit being stationary.

### 2.2 Further research plans

There are several avenues of research (some in the extension of my thesis) that seem promising to me, and that I would like to explore.

#### (1) Herr complex for crystalline and semi-stable representations

By a result of Kisin, the restriction induces a fully faithful functor from the category of crystalline representations of  $\mathcal{G}_K$  to that of  $p$ -adic representations of  $\mathcal{G}_{K_\pi}$ . As the latter are classified by the  $\varphi$ -part of the associated  $(\varphi, \tau)$ -module (*i.e.* the  $\tau$  is completely determined by  $\varphi$ ), it is natural to expect that the Galois cohomology of a crystalline representation should be described in terms of the associated étale  $\varphi$ -module alone. This is easy for the  $H^0$ , but the computation of  $H^1$  and  $H^2$  remain not clear so far. I believe that a satisfactory answer to this problem would be a very useful and important result.

In the special case of crystalline representations of Hodge-Tate weights 0 and 1, the answer should be even expressed in terms of the associated Breuil-Kisin module. In my thesis, I did construct a complex  $\mathcal{C}_{\varphi, \tau}(\mathfrak{M})$  doing this, but it requires to recover the action of  $\tau$ , which, heuristically, shouldn't be necessary.

## **(2) The case of finite flat group schemes**

In [5], Kisin explains how use his classification of  $p$ -divisible groups over  $\mathcal{O}_K$  to derive a classification of finite flat group schemes. If successful, the refinement of my results presented above would certainly give rise to the computation of Galois cohomology of finite flat group schemes over  $\mathcal{O}_K$ , which is a fundamental question as well.

## **(3) The relation of complexes $\mathcal{C}_{\varphi, \tau}$ with different choices of $\tilde{\pi}$**

It is interesting to ask the relations of the complexes  $\mathcal{C}_{\varphi, \tau}$  with different choices of  $\tilde{\pi}$ . Or more generally, how does the choice of  $\tilde{\pi}$  influence our theory?

## **(4) The case of $p = 2$**

As I have worked under the assumption that  $p \geq 3$ , it is natural to ask for the situation  $p = 2$ .

## **2.3 Applications to Iwasawa theory**

As in the classic case, the operator  $\psi$  I constructed for  $(\varphi, \tau)$ -modules is probably useful for an Iwasawa theory (probably still to be developed) within the framework of the Kummer extensions. I have plans to study Iwasawa theory to explore this direction of research.

## 2.4 $p$ -adic Hodge theory and Zink's display

One of the main objectives of  $p$ -adic Hodge theory is to find comparison theorems for (algebraic or even rigid) varieties defined over  $p$ -adic fields. Denote by  $C$  the  $p$ -adic completion of  $\overline{K}$ . By continuity, the group action  $\mathcal{G}_K$  acts on  $C$ . Let  $\chi : \mathcal{G}_K \rightarrow \mathbf{Z}_p^\times$  be the cyclotomic character. The first result in the field is due to Tate: if  $H$  is a  $p$ -divisible group over  $\mathcal{O}_K$ , there exists a canonical  $\mathcal{G}_K$ -equivariant isomorphism

$$\mathrm{T}_p(H)^\vee \otimes_{\mathbf{Z}_p} C \simeq \omega_{H^D}^\vee \otimes_{\mathcal{O}_K} C \oplus \omega_H \otimes_{\mathcal{O}_K} C(-1)$$

where  $\mathrm{T}_p H$  is the Tate module of  $H$ ,  $H^D$  the dual  $p$ -divisible group,  $\omega_H$  the tangent space at the origin and for  $j \in \mathbf{Z}$ ,  $C(j)$  denotes the field  $C$  with its  $\mathcal{G}_K$ -action twisted by  $\chi^j$  (Tate-twist). In particular, if  $A$  is an abelian variety over  $K$  with good reduction, there exists a  $\mathcal{G}_K$ -equivariant isomorphism:

$$\mathrm{H}_{\text{ét}}^1(A_{\overline{K}}, \mathbf{Q}_p) \otimes_{\mathbf{Q}_p} C \simeq \mathrm{H}^1(A, \mathcal{O}_A) \otimes_K C \oplus \mathrm{H}^0(A, \Omega_{A/K}^1) \otimes_K C(-1).$$

This led Tate to conjecture that if  $X$  is a proper and smooth scheme over  $K$  (or a rigid space over  $K$ ), there should be a "Hodge-Tate" decomposition, *i.e.* a  $\mathcal{G}_K$ -equivariant isomorphism

$$\mathrm{H}_{\text{ét}}^n(X_{\overline{K}}, \mathbf{Q}_p) \otimes_{\mathbf{Q}_p} C \xrightarrow{\sim} \bigoplus_{i+j=n} \mathrm{H}^i(X, \Omega_{X/K}^j) \otimes_K C(-j).$$

Since then, this conjecture has been refined by Fontaine, who constructed various  $p$ -adic period rings, that allow (under some hypothesis on  $X$ ) to state more precise comparison isomorphisms, involving extra structures (Frobenius, derivations, connections, *etc*) on the cohomology spaces. These conjectures have been proved by many authors (Fontaine-Messing, Kato, Faltings, Tsuji, Niziol, Andreatta-Iovita, Scholze) in various (and more or less general) contexts. In a remarkable work (*cf* [2]), Bhatt, Morrow and Scholze have recently developped an integral crystalline theory (which is all the more delicate as torsion phenomenon appear). The applications in Arithmetic Geometry are numerous (notably for automorphic forms theory and for the study of some moduli spaces).

As mentioned above, the most basic non trivial case is that of  $p$ -divisible groups. In that case, the crystalline comparison theorem relates the  $p$ -adic Tate module and the Dieudonné module of the  $p$ -divisible group (which are the incarnation of the  $p$ -adic étale cohomology and the crystalline

cohomology respectively). A very general Dieudonné theory has been developed by T. Zink (*cf* [8], [9]) and his collaborators: it allows the classification of  $p$ -divisible groups on very general affine basis in terms of "displays". These displays, which are computed in a rather direct way using Witt vectors rings, encode the Dieudonné module and its Hodge filtration. It is then tempting to reformulate the comparison theorem of a  $p$ -divisible group in terms of its display. The directions of research on this topic can proceed as follows:

**(1) Compute Galois cohomology of Tate module from associated display**

Based on the results in my thesis, I expect there are similar results to compute the Galois cohomology of the Tate module  $T_p(H)$  of a  $p$ -divisible group  $H$  from its display.

Thanks to the work of Zink and Lau, we know that the Tate module of a  $p$ -divisible group can be expressed using the associated Zink's display, while it is not clear how to compute its Galois cohomology with display (one problem is that the description of Tate module using display does not have a natural  $\mathcal{G}_K$ -action, but only a  $\mathcal{G}_{K_\pi}$ -action). Based on the work of the thesis, it might be possible to modify our complex  $\mathcal{C}_{\varphi,\tau}(\mathfrak{M})$  of Breuil-Kisin modules with the displays associated to the same  $p$ -divisible group, so as to compute the Galois cohomology of the Tate module of the  $p$ -divisible group in terms of its Zink's display.

**(2) Generalize the notion of display**

I would like to generalize the notion of display in order to go beyond the case of  $p$ -divisible groups: generalized displays should be able to encompass the (log-)crystalline cohomology and the Hodge filtration of proper and smooth schemes over  $K$  with good (or even semi-stable) reduction.

**(3) Comparison theorems**

As a long-term project, I would like to address comparison theorems (at least in the case of good reduction) in terms of generalized displays.

### 3 Possible local mentor

There are many professors in University of Milan that I might work with, for example it would be very helpful for me to work with professor Fabrizio Andreatta, who has particular interests on  $p$ -adic Hodge theory.

## References

- [1] C. Aribam and N. Kwatra, Galois Cohomology for Lubin-Tate  $(\varphi_q, \Gamma_{LT})$ -modules over Coefficient rings, arXiv preprint, (2019)
- [2] B. Bhargav, M. Matthew, P. Sholze: Integral  $p$ -adic Hodge theory, Publications mathématiques de l’IHÉS, **128**, (2018)
- [3] C. Breuil, Groupes  $p$ -divisibles, groupes finis et modules filtrés, Annals of Math. **151**, (2000), p.489-549.
- [4] X. Caruso, Représentations galoisiennes  $p$ -adiques et  $(\varphi, \tau)$ -modules, Duke Math. J. **162**, (2013), p.2525-2607.
- [5] M. Kisin, Crystalline representations and  $F$ -crystals, in Algebraic Geometry and Number Theory, Drinfeld 50th Birthday volume, Progress in Mathematics **253**, Birkhäuser, (2006), p.459-496.
- [6] B. Kupferer and O. Venjakob, Herr-complexes in the Lubin-Tate setting, arXiv preprint, (2020)
- [7] F. Tavares Ribeiro, An explicit formula for the Hilbert symbol of a formal group, Annales-Institut Fourier **61**, (2011), p.261-318.
- [8] T. Zink, A Dieudonné theory of  $p$ -divisible groups, in Class Field Theory - Its Centenary and Prospect, Advanced Studies in Pure Mathematics **30**, (2001), p.139-160.
- [9] T. Zink, The display of a formal  $p$ -divisible group, Astérisque **278**, (2002), p.127-248.
- [10] L. Zhao, Galois cohomology of  $p$ -adic fields and  $(\varphi, \tau)$ -modules, PhD thesis, (2021)